

Polar Motion and Earth Tides from Laser Tracking [and Discussion]

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Polar motion and Earth tides from laser tracking

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The tracking of near-Earth satellites with laser systems permits the determination of the variation of latitude of the tracking station and the variation in the rotation of the Earth. The present-day capability of a single station is approximately 75 cm in latitude averaged over 6 h and 0.8 ms in the length of day. When the Laser Geodynamics Satellite (Lageos) is launched, a network of laser stations is projected to be able to achieve better than 10 cm in each coordinate from less than one day of tracking. The perturbations of near-Earth satellites by solid Earth and ocean tides are now measurable and can provide new information about the Earth and oceans. The orbit perturbations have long periods (days, months) and the analysis of orbital changes are providing estimates of the amplitudes and phases of the major tidal components.

INTRODUCTION

Over the past decade laser tracking of near-Earth satellites has developed to the stage that N.A.S.A. laser systems are now capable of measuring the distance to specially equipped satellites with an accuracy of about 5 cm. This capability provides the basis with which parameters of geodetic and geophysical interest, such as polar motion, Earth rotation and tides, may be measured. Since 1970 N.A.S.A. Goddard Space Flight Center has been conducting experiments and developing techniques to obtain and analyse these precise data. Until recently only two N.A.S.A. laser tracking systems were available for this work, one of which was primarily for system research and development, thus the major thrust has been to establish techniques with a single system that might be logically extended as further systems become available. At the present time N.A.S.A. Goddard Space Flight Center has three operational systems, two of which are mobile, a third mobile system will be completed in 1976 and by late 1977 a further five mobile systems will be available, making a total of nine systems. Together with four systems of the Smithsonian Astrophysical Observatory, they will constitute N.A.S.A.'s global laser tracking network.

In this paper we will describe the techniques that are available for measuring polar motion, Earth rotation and earth tides with laser tracking of satellites, summarize the results of our experiments so far and discuss the future prospects when all the tracking systems are completed and the Laser Geodynamics Satellite (Lageos) has been launched.

POLAR MOTION AND EARTH ROTATION

From a single point on the Earth's surface it is only possible to measure one component of polar motion, that is, variation of latitude, and the length of day. Conventional techniques measure these parameters by observing their apparent motion with respect to the stars; the satellite

method employs the monitoring of Earth motion with respect to the orbit of a satellite. Thus, the stability of the orbit of the satellite and a knowledge of the orbit perturbations are essential in this method, as well as an accurate method of tracking.

The first task in the measurement of variation of latitude at a single tracking station is to establish a reference orbit. Tracking data on a particular satellite over a period of time, say one month, is used for this purpose. If the orbit is well determined and the major perturbations, such as Earth and luni-solar gravity, are calculated with considerable accuracy then this reference orbit is usable for an extended period (at least one year). Movement of the tracking station is determined from a comparison of the orbit of the satellite over a short period, say 12 h, with the reference orbit. In principle, all the orbit parameters in the short (12 h, say) orbital arc can be compared with the reference orbit parameters for the same period but in reality the most important parameter for comparison is the orbital inclination. The primary reason is that the inclination is a very stable parameter over very long periods of time and can, by careful observation, be determined from observations with very high accuracy. Furthermore, the orbital inclination in the short arc is directly related to the latitude of the tracking station when only one tracking station is used in the orbit determination. Thus, changes in the latitude of the tracking station are reflected into the orbital inclination of the short arc which can be identified when compared with the inclination in the very long arc.

The orbital inclination of a near Earth satellite is continuously changing under the action of perturbing forces so a complete understanding and modelling of these forces is essential for the computation of the very long reference orbit. Any errors in this modelling will show up as an apparent latitude variation of the tracking station. To some extent this is true for the very short arc except that station coordinate errors (particularly latitude) are more important than almost any perturbing forces. The limiting factor in the 'shortness' of the short arc is the strength of the orbit determination process using only range measurements. For example, for a 10 min pass of a near Earth satellite, range measurements alone from a single site are unable to provide an orbit. Two consecutive passes (approximately 1 revolution apart), of the satellite observed from the same site are adequate but the determination is not very strong with the orbital inclination having a standard deviation of about $0.1''$ when the data accuracy is 10 cm and the data rate one range measurement per second. Our experience to date with the Beacon Explorer C spacecraft indicates that four consecutive passes covering a 6 h period provide very strongly determined orbital inclinations at better than 10^{-3} arcsecond but that errors in our model of the Earth's gravitational field are considerably larger, thereby limiting our effective present-day capability.

In order to determine the orbital inclination as accurately as the observations will permit, the tracking station is best located near the northern or the southern apex of the orbit. At this position the spacecraft is travelling west to east (or east to west) and the tracking station is observing the satellite in a northerly (or southerly) direction. The distance of the satellite from the station is measured very precisely by the tracking system and because the satellite is being observed near the position of maximum latitude (equal to the orbital inclination) the orbital inclination is very well determined. Figure 1*a* shows the orbit and station configuration for the measurement of variation of latitude from a single tracking station using a medium inclination orbit.

An analogous situation exists for measuring Earth rotation from a single station. Instead of measuring the earth rotating with respect to the stars, we measure the Earth rotating within the orbit of a satellite. The optimum orbit for this measurement is a polar orbit so that the tracking station observes the spacecraft to the east or west. In this way the range measurements can

provide a precise determination of the time and longitude of the satellite as it crosses a particular latitude, say the latitude of the tracking station. Daily observations of the time and longitude of the satellite crossings provide a measure of the rotation of the Earth with respect to the orbit plane of the spacecraft. Because the orbit plane of the satellite precesses due to the perturbing forces, corrections must be applied to the results. This is accomplished by deriving a long orbital arc in which all known perturbing forces are included and comparing the daily time and longitude crossings not with each other but with the long reference orbit. The orbit and station configuration for single station Earth rotation measurements are shown in figure 1*b*.

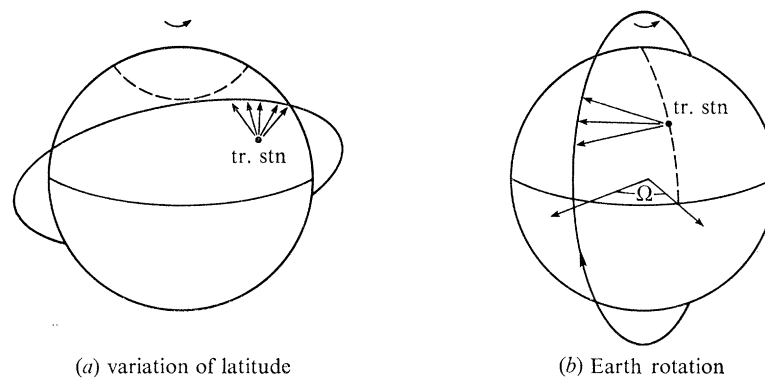


FIGURE 1. Measurement of variation of latitude and Earth rotation from a single satellite tracking station.

There are obvious limitations on the single station technique for measuring latitude or Earth rotation. The most important of these is that the measurements are not related to an inertial reference frame and are therefore unable to separate true polar motion from incorrect modelling of precession and nutation. However, since the latter is much better known than polar motion the primary limitation on the technique becomes the long-term stability of the orbit. The node of the spacecraft orbit precesses due to the Earth's oblateness, other gravity terms, and other perturbing forces so there will probably always be an uncertain nodal acceleration of some magnitude in the spacecraft orbit which will manifest itself as an off-set in the rotation rate of the Earth. Thus variations in rotation rate will be the most promising avenue of research from this technique. Studies that have been performed on this technique indicate a precision in the length of day of a few tenths of a millisecond.

The orbital inclination of a high altitude spacecraft does not, in general, undergo any secular changes thus the measurement of variation of latitude does not have the same inherent problem that arises for the measurement of Earth rotation. The orbital inclination undergoes periodic variations ranging from a few tens of minutes in time to a few months, and even years. In general, the shorter the period the smaller the amplitude, with the larger perturbations (such as luni-solar gravity) being of the order of $10''$. Fortunately, these larger perturbations are known very precisely so that the limiting factors in measuring polar motion are generally the short period gravity perturbations.

An experiment (Smith *et al.* 1972) conducted in 1970 which employed this technique gave a 1 m latitude capability from 6 h of laser tracking of Beacon Explorer C (BE-C). In a 5-month period the latitude of a laser at G.S.F.C. in Greenbelt, Maryland was observed to change by about 10 m and in a comparison with the smoothed Bureau International de l'Heure values an

r.m.s. deviation of slightly less than 1 m was obtained. At the time that the 1970 experiment was conducted the accuracy of the laser system was estimated to be 50 cm.

Recently, a 3-week period of data in the 1970 experiment has been re-analysed by using a slightly modified approach (Dunn, Smith & Kolenkiewicz 1976). A reference orbit was determined for the three week data period and subsequently the location of the tracking station on each of 13 days during the period was adjusted to best fit the reference orbit. Latitude and time were recovered from the data. Figure 2 shows the result for this period; the latitude and time variations are the residuals to the computed B.I.H. values and have r.m.s. deviations of 74 cm and 0.81 ms. The time variation represents the difference in time of arrival of the satellite according to the observations and the reference orbit. Thus the time residuals are along-track errors in the satellite orbit plus errors in station longitude arising from errors in the B.I.H. model of the Earth's rotation. Since the along-track position of a satellite is the most sensitive orbit parameter to perturbing forces we believe the time residuals probably reflect errors in the satellite motion rather than Earth rotation. However, they probably indicate an upper limit on errors in the latter for this time period.

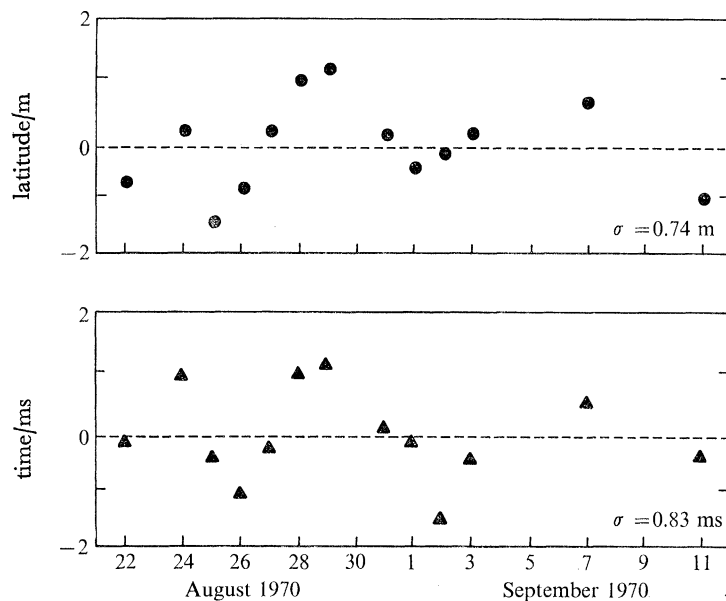


FIGURE 2. Variation of latitude and time at the laser tracking site in Greenbelt, Maryland.

To assess more fully the potential of deriving variation of latitude from a single station, a simulation of a 150-day experiment based on using the Laser Geodynamics Satellite (Lageos) has been completed. Lageos, due for launch shortly [launched on 4 May 1976] is a spherical satellite covered with laser retro-reflectors and will be placed in a high altitude orbit of nearly 6000 km. Table 1 gives the spacecraft details and orbit which have been designed to utilize laser ranging methods for the measurement of geodetic and geodynamic parameters. Our simulation estimates the effect of errors in the force model and the location of the tracking station on deriving latitude variations. The results, shown in table 2 for models expected to be available in 1976-7, show that for short orbital arcs of about 12 h a capability of about 16 cm is anticipated, while for a 4.5 day arc a result near 3 cm is expected. In addition to the error sources shown in table 2, we also considered the effects of solar radiation pressure and air drag on the orbit, and body tides on

the position of the tracking station. Neither of these effects contributed as much as 1 cm to the error based on an error of 10 % in our models for these effects. As a result of these simulations and our experimental results with low altitude spacecraft, such as BE-C, we feel reasonably optimistic that 5–10 cm variation of latitude from half a day to several days will be achievable in the near future. Further details of these techniques can be found in an earlier paper (Smith, Kolenkiewicz, Agreen & Dunn 1973 *b*).

TABLE 1. LASER GEODYNAMICS SATELLITE (Lageos)

sphere: 60 cm diameter	orbit: circular
426 laser retro-reflectors (visible wavelength)	5900 km altitude
4 laser retro-reflectors (far infrared)	110° inclination
411 kg, aluminium, brass	

Launch Date: 4 May 1976

TABLE 2. LAGEOS VARIATION OF LATITUDE SIMULATION AT A SINGLE STATION
ERROR SOURCES

(GM, 1 part in 10^7 ; gravity, 1/12 (Standard Earth 1 – APL 3.5); station coordinates, 1 m.)

	sources of error (cm)				RSS
	GM	grav. field	station lat.	station ht.	
0.5 day orbital arc	1	14	4	7	16
2.5 day orbital arc	1	4	1	2	5
4.5 day orbital arc	1	2	1	1	3

The methods just described can be equally applied to a network of stations all working independently with the resulting output being both components of polar motion with accuracies equal to approximately $\sqrt{(2/n)}$ times the single station capability, where n is the number of stations in the network. However the existence of a network has greater advantages than just providing both components and increasing the accuracy through additional measurements. If we suppose that the locations of the stations in a network are perfectly known in some coordinate system then it is possible to derive the pole position (in the same coordinate system) about which all the stations are rotating from the observation of a satellite over a period of a few hours. Figure 3 shows a small network of stations tracking a satellite at two different times. Because the same relative coordinates of the stations apply for both tracking configurations it is possible to solve for the orbit of the satellite simultaneously with the (average) coordinates of the point about which the tracking network is rotating during the total period of the data. There are two main advantages of this technique, together with advantageous consequences. The first is that a reference orbit is no longer required and hence the limiting capability is the ability to determine a good orbit through a few hours (or days) of data rather than having to account for perturbations over months or years as is necessary in a long arc; and further, the pole position is independent of the satellite orbit and hence different satellites can, in principle, be used from one day to the next. However, systematic errors involving the satellite could complicate this position. The second main advantage is that orbital perturbations and precession and nutation model errors are separable from the pole position that is derived although they are not themselves separable. That is to say, that precession and nutation errors do not change the relative position of the pole and the network, only the relative position of the orbit and the network – which is exactly the same as an orbital

perturbation. In the case of two stations on opposite sides of the pole, they see the same effects due to orbit inclination perturbations and precession and nutation but the opposite effect when the pole moves toward one of them (to first order) to the extent that there are no unknown perturbations during the observation period. It is for the latter proviso that the satellite method of deriving polar motion works best over short averaging times – as the data span increases so does the probability of orbit error.

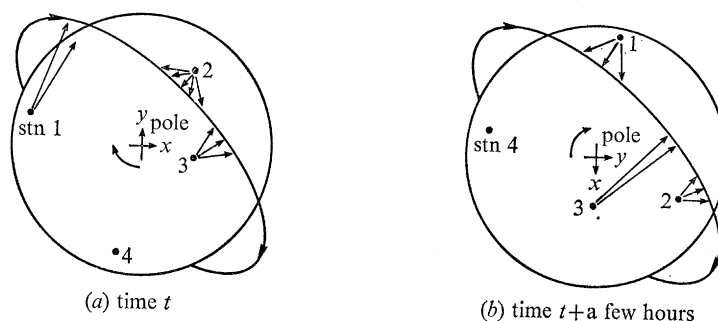


FIGURE 3. Determination of polar motion from satellite tracking network. At time, t , stations 1, 2 and 3 track the satellite. At time $t + a$ few hours the Earth has rotated so that the stations track the satellite on a different trajectory. Analysis of the data from both times enables the point about which the stations are rotating (the pole) to be determined.

The only satellite tracking network routinely deriving pole position data is that of the U.S Navy which utilizes Doppler tracking (Anderle 1973). This network has been deriving pole positions since the latter part of the last decade to an accuracy of about $0.02''$ with an averaging time during most of this period of two days and during this time have used several different satellites. Although the Doppler satellites are low (about 1000 km) compared to Lageos, the system is not dependent on the long-term stability of the orbit – only on orbit stability over the two day period. By comparison with the Doppler, the laser is a much more precise tracking system and the Lageos spacecraft is far superior to any low-altitude one. Thus we expect the success of the Doppler network to be comfortably exceeded by a comparable laser network employing Lageos, weather permitting. Our projection of the ultimate accuracy of the laser network to determine the pole position is approaching 10^{-3} arcsecond over averaging times of 12–24 h; but we shall have to wait until nearer the end of this decade before this is realized.

EARTH TIDES

One of the larger perturbations of long period that affect the orbit of an Earth satellite are those caused by the tides, both solid-earth and ocean. The Sun and Moon both raise tides on the Earth of a few tens of centimeters amplitude which to a satellite orbiting the Earth appear as a time varying component of the Earth's gravitational field. A near Earth satellite circles the Earth every 100 to 120 min and consequently samples the tidal distortion of the Earth some thirteen or fourteen times per day, depending on the satellite's period of revolution. Each of the revolutions traces an almost identical path with respect to the tidal bulges of the Earth, changing only due to the movement of the Sun and the Moon in space and the precession of the orbit of the satellite. The tidal perturbing forces produce high frequency perturbations of the satellite orbit of approximately half a revolution in period, which are too small to observe, and also low frequency

perturbations that arise from the relatively slow motions of the orbits of the satellite, Sun and Moon. It is these longer period terms that produce the large perturbations.

The principal tidal terms of the solid-earth have periods near 12 h and 24 h because the length of the day is 24 h. If the Earth's rotation rate were to decrease so that the day lengthened then the principal tidal periods would be accordingly modified. The principal tidal terms that affect a satellite orbit are governed (partly) by the rotation of the orbit, not the Earth, which typically precesses at a few degrees per day. This rate of motion is comparable to the Sun's motion and considerably less than that of the Moon. Thus tidal periods seen by satellites are typically tens of days, months, or even years, depending on the orbit. Table 3 gives the arguments, both for the Earth and satellite, for six principal waves O_1 , K_1 , P_1 , M_2 , S_2 , K_2 . The satellite arguments are obtained by replacing the sidereal time by the right ascension of the satellite node in each of the Earth tide arguments.

TABLE 3. PRINCIPAL TIDAL WAVES

symbol	Earth argument	satellite argument
O_1	$\theta - 2s$	$\Omega - 2s$
K_1	θ	Ω
P_1	$\theta - 2h$	$\Omega - 2h$
M_2	$2(\theta - s)$	$2(\Omega - s)$
S_2	$2(\theta - h)$	$2(\Omega - h)$
K_2	2θ	2Ω

θ , Sidereal time.

s , Mean longitude of the Moon.

h , Mean longitude of the Sun.

Ω , Right ascension of node of satellite orbit.

Theories for the tidal perturbations of satellite orbits have been published by several authors, including Kozai (1968), and Musen & Felsentreger (1973). Their theories enable the theoretical amplitude of each tidal component on the orbit to be computed, an example of which is shown in figure 4 for the inclination of the Beacon Explorer C (BE-C) satellite. In these calculations the value of Love's number of second degree, k_2 , was assumed to be 0.3. Only the major tidal terms are shown with amplitudes larger than 0.05". In this figure we see the K_1 tide has the longest period and the largest amplitude, while M_2 has the shortest period at a little over 10 days. The separation of the tidal waves shown in figure 4 is typical for a near Earth satellite, and no two satellite tidal spectra are the same – unless their orbits are the same.

The first observations of Earth tide perturbations of satellite orbits were by Kozai (1968) and Newton (1968) and subsequently by Smith, Kolenkiewicz & Dunn (1973*a*) and Douglas, Klosko, Marsh & Williamson (1974). In the later works it became clearly evident that the amplitudes of the tidal perturbations were different from the theoretical values and that the disagreement depended on the orbit of the satellite. Our own work has used laser tracking of the Beacon Explorer C spacecraft and our initial investigation used data over a five month period in 1970 (Smith *et al.* 1973*a*). In this analysis we were able to demonstrate that the tidal perturbation of the BE-C inclination could be represented by a tidal potential of second degree with an equivalent Love number of

$$k_2 = 0.245 \pm 0.005$$

and a phase lag (ϕ) of

$$\phi = 3.2^\circ \pm 0.5^\circ.$$

At the time we felt that the complete tidal signature had been removed from the inclination data by a simple adjustment of the theoretical amplitude and the introduction of a small phase lag.

No consideration in the analyses by us or other workers had been given to the effect of ocean tides on the satellite orbit and subsequently Lambeck, Cazenave & Balmino (1974) showed that this was probably the explanation for the orbit-dependent tidal amplitudes that were being obtained. They further showed that the ocean tidal perturbations would be frequency dependent and therefore that the recovery of a single Love number for a satellite orbit would be ultimately inadequate, even if it were valid for the solid Earth, which was probably not the case.

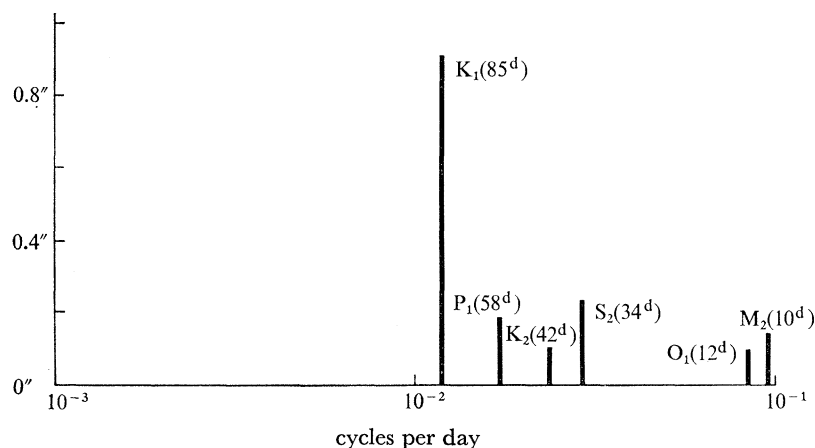


FIGURE 4. Theoretical spectrum of BE-C tidal perturbations in inclination.

TABLE 4. ESTIMATED TIDAL PARAMETERS FROM BEACON EXPLORER C

symbol	Love number	phase lag
O_1	0.27 ± 0.08	—
K_1	0.25 ± 0.01	$5.7 \pm 1.6^\circ$
P_1	0.20 ± 0.04	—
M_2	0.21 ± 0.05	—
S_2	0.26 ± 0.03	—
K_2	0.34 ± 0.06	—

The BE-C laser orbits have recently been re-analysed (Rubincam 1976). A slightly larger data set covering a seventeen month period has been used and a separate effective Love number and lag angle for the whole Earth has been obtained for each of the principal tidal components. One of the preliminary solutions is shown in table 4. In addition to the amplitudes, phase lags were also estimated but only one, that for the K_1 tide, was significantly different from zero. No attempt was made to separate the solid Earth effects from those of the ocean and atmosphere. The values in table 4 show the general tendency for a smaller overall value for k_2 as indicated by the earlier analysis when only a single value was recovered. The accuracy of the numbers in table 4 is not very great due to the limited number of orbits (thirty-six) used in the analysis. However, the values for K_1 and P_1 appear to be significantly different from the generally accepted value of 0.3 for k_2 and presumably represent the influence of the oceans, already discussed. An additional assumption that has been made in this analysis is that the tidal potential is purely of degree 2. For the solid earth this is probably a reasonable assumption but may not be true for the oceans and,

if so, Lambeck *et al.* (1974) have shown that second degree Love numbers will have an apparent orbit dependence due to the effects of degrees 4, 6, ..., etc. Further, it is impossible to separate second and fourth degree tidal terms in data on a single satellite. Thus a combined solution will be necessary for full separation of the tidal terms in both frequency and degree. Tidal signals have already been measured in the Geos 1 and 2 satellites (Felsestreger, Marsh & Agreen 1976) as well as BE-C and soon we can expect results from Starlette and Geos-3, followed in a year or two by Lageos. A substantial data base is thus being obtained for future studies. Finally it is important to mention the potential accuracy of the future solutions. Tidal perturbations of the orbital inclination of a near Earth satellite can be as large as a few seconds of arc, about 100 m in satellite position, for the major terms. Precision tracking is already at the few centimetre level and orbit determination accuracy of the inclination is at a few hundredths of a second of arc, based on our ability to observe polar motion. Even on satellites at about 1000 km altitude, such as BE-C and Geos 1, 2 and 3, a potential of 5×10^{-3} arcsecond in inclination accuracy can be expected in the future giving an effective signal to noise ratio of at least 200 : 1 for the major terms. From high precision tracking of several satellites, a combined tidal solution will probably provide reliable values for the eight to ten larger terms with accuracies of the order of a few tenths of one percent in amplitude.

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Discussion

C. A. MURRAY (*Royal Greenwich Observatory*). What exactly is the reference frame to which the Earth rotation components, derived from satellite laser tracking, are referred?

Is it the star catalogue system (nominally FK 4) used in deriving U.T. 1 and polar motion from astrometric observations, or is it the system defined by the dynamical equinox and equator?

D. E. SMITH. For a single station, operating independently, the variation of latitude measurements are referenced to the orbital ephemeris of the satellite, which is defined in terms of the equator and equinox of date. For a network of stations, the components of pole position are measured with respect to the relative locations of the ground stations in a centre-of-mass coordinate system.

J. A. WEIGHTMAN (*Geodetic Office, Elmwood Avenue, Feltham, Middlesex*). It was pointed out by Professor Markovitz some years ago that one considerable advantage of siting all the international latitude stations at the same geographical latitude, for the astronomic observation of polar motion, was that the result was quite independent of star catalogue error, since the 'position lines' for the pole merely touched a larger or smaller circle; could one argue analogously that some types of error would be eliminated by siting the laser stations for observing polar motion along a single parallel of latitude?

D. E. SMITH. Yes, the same concept applies for a small network of tracking stations. The effect on the measurement of Earth rotation of errors in computing the satellite orbit is reduced for stations on similar latitudes.